Please answer all 3 questions and subquestions below.

Problem 1

(a). In section 8.1.3 we make the assumption that informed trading is more correlated than uninformed trading, whereas in section 9.5 the opposite argument is made. In which circumstances do you think informed trading is more correlated, and in which circumstances do you think noise trading is more correlated? What effect do you think it would have on the analysis of section 8.1.3 if we used the assumption that noise trade is more correlated?

Suggested solution. The assumption that informed trading is more correlated seems more pertinent in market where there is plenty of information, for instance markets where information can be bought from analysts or where fast trading is prevalent, such that fast traders use public information to quickly trade before prices adjust. Since in these two instances traders will be reacting to the same information, it seems likely that their trades are correlated. On the other hand, as the book suggests, macroeconomic shocks may correlate liquidity needs.

The analysis in 8.1.3 should be relatively unchanged: what is important is that informed and noise traders have *different* correlations in trade.

(b). On page 143 of the textbook, the authors discuss the multi-period version of Kyle (1985). They note that in the multi-period version, "the informed trader trades less aggressively than in the one-period case, in order to avoid dissipating his information advantage too quickly." Why so? Suppose the insider has information that the value is high – would he not be better off trading on this information quickly before the market maker realizes it?

Suggested solution. Roughly speaking, splitting the trade over several periods corresponds to 'hiding behind' more noise traders. Since noise traders will most often have moderate (positive or negative) demand due to the normal distribution, several periods of moderately high demand seems less conspicuous than one period of very high demand. Therefore, the aggregate price effect is less.

(c). Using the insights from Bikhchandani and Sharma, explain why herding due to informational cascades may occur (you may want to use the example we analyzed in class). Why does the price mechanism help to prevent herds.

Suggested solution. Herding occurs when public information becomes sufficiently strong such that agents no longer pay attention to private information. But when this happens, learning stops. Each agent will only learn his own private signal, which he will ignore, and nothing else. He has no private incentive to act on his own information. The price mechanism essentially provides him with this private incentive, since good information will always be worth something, unlike when the price is fixed.

Problem 2

Let us consider an extension to the Kyle (1985) model of lecture 5. Suppose as before that there is a single asset with value $v \sim N(\mu, \sigma_v^2)$. An *insider* observes the true value and places a market order x. However, we will now suppose that the insider has an *initial holding* w_0 of the asset, such that conditional on price p, his final wealth is

$$w = (w_0 + x)(v - p).$$

Furthermore, we assume that the insider is *risk averse*, and in particular that he has mean-standard deviation preferences, i.e. he maximizes

$$U = \mathbb{E}[w] - \rho\mathbb{S}(w),$$

where $\mathbb{S}(w) = \sqrt{\mathbb{V}(w)}$ is the standard deviation of w, and $\rho > 0$ is a risk-aversion parameter.

Apart from the informed trader, there is a single *noise trader* with demand $u \sim N(0, \sigma_u^2)$. Total demand is thus $q = x + u$. There is a single *competitive* market maker, who observes total demand and sets prices at expected values, i.e. $p = \mathbb{E}[v|q]$. The variables u and v are jointly normal and independent.

(a). Suppose the market maker sets prices according to $p = p_0 + \lambda q$, where $\lambda > 0$ and p_0 are parameters. Consider then the insider. To simplify matters, suppose that w_0 is sufficiently big such that in equilibrium, $w_0 + x \geq 0$.[1]

Solving his optimization problem, show that he follows the strategy

$$x = \beta(v - x_0),$$

where $\beta = \frac{1}{2\lambda}$ and $x_0 = p_0 + \lambda(w_0 + \rho\sigma_u)$.

Solution. Plugging in the pricing rule, we get

$$U = (w_0 + x)(v - p_0 - \lambda x) - \rho\lambda|w_0 + x|\sigma_u.$$

If $w_0 + x > 0$ this gives the FOC

$$v - p_0 - \lambda(w_0 + 2x) - \rho\lambda\sigma_u = 0.$$

Solving this for x gives the desired answer.

(b). Conditional on the insider strategy that you found in the previous question, use the result on normal distributions from the slides/book[2] to show that

$$\mathbb{E}[v|q] = \mu + \alpha(q - \beta(\mu - x_0)),$$

where $\alpha = \frac{\beta\sigma_v^2}{\beta^2\sigma_v^2 + \sigma_u^2}$.

Solution. Using the Normal distribution lemma:

$$\mathbb{E}[v|q] = \mathbb{E}[v] + \frac{\mathbb{C}(v,q)}{\mathbb{V}(q)}(q - \mathbb{E}[q]) = \mu + \frac{\beta\sigma_u^2}{\beta^2\sigma_u^2 + \sigma_v^2}(q - \beta(\mu - x_0)).$$

[1] Clearly this will not always hold, but we make this assumption to analyze the case of a 'rich' insider.

[2] In particular, footnote 4 on page 136. You may also want to refer to Problem Set 1.

(c). Argue that β and λ is the same as in the model we saw in the lecture. Which of the assumptions we made implies that risk-aversion in this model does not change the insider's aggressiveness?

Solution. From the optimal strategy of the insider and the market maker's pricing rule we obtain $\beta = \frac{1}{2\lambda}$ and $\lambda = \alpha$, the exact same conditions as in the lecture. Thus $\lambda = \frac{\sigma_v}{2\sigma_u}$ and $\beta = \frac{\sigma_u}{\sigma_v}$.

Because we have assumed mean-standard deviation preferences, risk-aversion has a discrete effect: it causes a jump in the optimal strategy at the cutoff point $x + w_0 = 0$. But since we have assumed that w_0 is large, this jump is assumed away. Thus, risk-aversion affects neither price sensitivity nor trader aggressiveness, but it does have a level effect on trading.

(d). Find x_0 and p_0. How do they compare to their values in the lecture? What is the intuition for the difference?

Solution. We find $\alpha = \frac{\sigma_v}{2\sigma_u}$, $x_0 = p_0 + \frac{\sigma_v}{2\sigma_u}(w_0 + \rho\sigma_u)$ and $p_0 = \mu - \frac{\sigma_v}{2\sigma_u}\frac{\sigma_u}{\sigma_v}(\mu - x_0) = \frac{\mu + x_0}{2}$. Solving this gives $x_0 = \mu + \frac{\sigma_v}{\sigma_u}(w_0 + \rho\sigma_u)$ and $p_0 = \mu + \frac{\sigma_v}{2\sigma_u}(w_0 + \rho\sigma_u)$.

In the lecture, both these values were μ. Thus, risk aversion on part of the insider increases the equilibrium price for any demand level, and lowers the equilibrium demand for any value. As the insider is a net holder of the asset and is risk averse, he is less wiling to invest in more, explaining the drop in demand. As average demand will be lower, high demand is a stronger signal of informed trading, explaining the price increase.

(e). Suppose now that the insider has mean-variance preferences

$$U = \mathbb{E}[w] - \rho\mathbb{V}(w).$$

Suppose again that the insider follows a linear strategy $x = \beta(v - x_0)$ and the market maker uses a linear pricing rule $p = p_0 + \lambda q$. Furthermore, suppose $w_0 = 0$. Find the equilibrium conditions without solving them, i.e. find β as a function of λ, and λ as a function of β. Plot the two functions for some parameter values and guess as to how you think the equilibrium values of β and λ will compare to what we saw in class.

Solution. Now, substituting the price rule we get

$$U = x(v - p_0 - \lambda x) - \rho x^2 \lambda^2 \sigma_u^2.$$

Take the FOC with respect to x:

$$v - p_0 - 2\lambda x - 2\rho x \lambda^2 \sigma_u^2 = 0.$$

This gives

$$x = \frac{v - p_0}{2\lambda(1 + \rho\lambda\sigma_u^2)}.$$

On the other hand, we have

$$p = \mathbb{E}[v|q] = \mu + \alpha(q - \beta(\mu - x_0)).$$

where α is as before.

Thus, $\lambda = \alpha$ as before but $\beta = \frac{1}{2\lambda(1+\rho\lambda\sigma_u^2)} < \frac{1}{2\lambda}$ for all $\lambda > 0$. Plotting for $\sigma_u^2 = \sigma_v^2 = \rho = 1$ (see Figure 1 below) gives the impression that this will lead to lower β and λ (transposing the positive-valued blue line south-west will lower equilibrium values on both axes). That is, risk-aversion will make the insider less aggressive in equilibrium. This seems natural given $w_0 = 0$, since aggressiveness implies taking risky positions. Furthermore, the price will be less sensitive to trade. This seems as a natural equilibrium response to less aggressive insider trading.

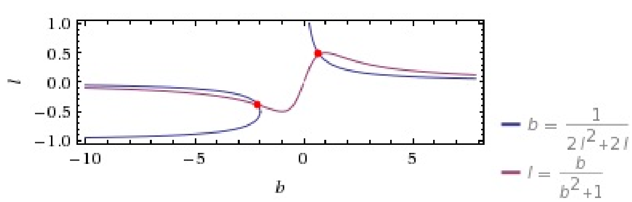

Figure 1: Best-response plot

(f). Find the equilibrium value of β in two cases: (i) $\sigma_u^2 = \sigma_v^2 = \rho = 1$, and (ii) $\sigma_u^2 = \sigma_v^2 = 1$ and $\rho = 2$. What is the intuition for the difference in trader aggressiveness in the two cases?

Solution. Solve

$$\lambda = \frac{\beta\sigma_v^2}{\beta^2\sigma_v^2 + \sigma_u^2}$$

and

$$\beta = \frac{1}{2\lambda(1 + \rho\lambda\sigma_u^2)}$$

for β and plug in $\sigma_u^2 = \sigma_v^2 = 1$ to get

$$\beta = \frac{(1+\beta)^2}{2\beta(1 + \beta\rho + \beta^2)}$$

Solving for β when $\rho = 1$ and taking the positive real solution gives $\beta \simeq 0.72$, whereas solving for β when $\rho = 2$ and taking the positive real solution gives $\beta \simeq 0.60$. Thus, higher risk aversion seems to lower equilibrium aggressiveness.

Problem 3

Below is an article from the Financial Times on May 5, 2015. Please write a short essay discussing to which extent the course readings can relate to the issue of this text. In particular, consider the theories exposed in lecture 9, but feel free to include theories from other parts of the course (for instance theories of market making). Also, you are welcome to elaborate your answer beyond the syllabus.

"You may be surprised by what could end up mattering most on the potential implications for financial markets of the upcoming UK general election.

Yes, markets might react, at least initially, to what they currently perceive as different outcomes for economic policies and EU membership depending on who ends up governing and in what form. But a more lasting differentiator is likely to come from a less appreciated factor – that is, the various approaches towards a financial services industry that has yet to restore its credibility after the debacles of the global financial crisis.

At least superficially, the Conservative and Labour party manifestos propose different futures for economic management. Yet their ability to implement their paths is subject to considerable political and economic constraints.

(...)

Rather than major changes to economic management, what is likely to follow in most post election scenarios is just some tweaking of the current approach. (...) The gap appears larger when it comes to the EU. As the Conservatives have promised to hold a referendum on staying in Europe if returned to Downing Street, a win for them would initially bring greater uncertainty into the marketplace. By undermining UK companies' access to such a large market, an exit would weaken corporate profits. (...) Yet, it is highly unlikely that such a referendum would lead to a UK exit.

Indeed, the biggest potential difference, and the one with the greatest potential financial market impact, is elsewhere.

A Labour government would be likely to take a less lenient approach towards a financial services industry that has yet to overcome a huge trust deficit within society. It would be more open to tighter regulation, limits on pay and the pursuit of a series of high profile legal cases against offending companies. It would engage in a spirited debate with companies threatening to move their headquarters from the UK. It would also be less inclined than the Tories to fight off creeping European regulatory infringement on the operations and risk taking of financial institutions.

With all this leading to a further shrinkage of the financial sector, markets would price in higher risk premiums for both bonds and equities on account of lower future liquidity – that is, tighter constraints on broker-dealers assuming significant counter-cyclical risk as end investors wish to reposition themselves amid changes to fundamentals elsewhere. Such an adjustment could be quite pronounced given the extent to which markets, captivated by the illusion of liquidity, have grossly underpriced a risk factor that is subject to both secular and structural deterioration. It would then be more incumbent on the government to deliver on the measures needed meaningfully to boost growth and validate high financial asset prices, including productive infrastructure investment and tax reform. The alternative is asset prices that converge to the lower fundamentals, overshoot them and risk contaminating the general economy.

In regard to the upcoming election, financial markets have less to worry about than commonly thought on traditional economic and EU issues. What they should be doing instead is better guarding against the gross underpricing of liquidity risk, an already pronounced phenomenon that could prove an important differentiator when assessing the implications of the polls."

Suggested solution. The article offers several issues to discuss.

- An election is an uncertainty event, and such uncertainty is priced in the market. Models such as the one by Stoll on risk-averse market makers explain how this occurs.
- The main point of the article is that the biggest change in policy, seen from a financial sector point of view, is going to be in financial regulation, with a Labour government more likely to bring about changes that would shrink the financial sector. Again, the Stoll model explains how dealers take on risk. If their trading costs rise as a result of regulation, they may not be willing to do so any more, diminishing liquidity. The Glosten model of limit orders also directly links liquidity to the cost of posting limit orders. Higher costs of trading and higher extraordinary costs, such as legal costs, will also be passed on directly to spreads, as suggested by the analysis in lecture 4.
- All the aforementioned factors will lower the liquidity of markets, and the theories of lecture 9 can be used to explain the depressive effect on prices, in particular the sections relevant to liquidity risk.
- The article then makes the point that the government must boost 'fundamentals' by inducing economic growth, since otherwise there will be a downwards adjustment that can lead to 'overshooting' and contamination. Overshooting may be an effect of the two theories we covered in lecture 13: either herding or uncertainty about the size of the contraction.